several authorities on these facts, and some curious allusions to them in ancient Irish manuscripts.*

Dr. A. S. Hart read a paper on the form of geodesic lines through the umbilic of an ellipsoid.

If ω be the angle at the umbilic of an ellipsoid, between the principal section of the surface and any other geodesic line, and if θ be the angle between the plane of the principal section through the umbilics and the osculating plane of this geodesic line, at any point A, and if α be the semi-angle of the right cone circumscribing the ellipsoid at the point A, a, b, and c being the semi-axes of the ellipsoid, the angle θ may be determined by the following equation:

$$\frac{\tan \frac{1}{2}\theta}{\tan \frac{1}{2}\omega} = e^{\sqrt{(a^2-b^2)}\sqrt{(b^2-c^2)}\int_{\frac{\pi}{2}}^{\alpha} \frac{d\alpha}{\sqrt{(a^2 \tan^2\alpha + b^2)}\sqrt{(c^2 \tan^2\alpha + b^2)}}}.$$

Hence it follows that, as this line passes and repasses for ever through the two opposite umbilics, the tangents of the halves of the angles which it makes at these points with the plane of the umbilics will be a series of continued proportionals, the coefficient of the common ratio being determined by making $\alpha = -\frac{\pi}{2}$ in the above equation.

If $c = 0$, the ellipsoid becomes a plane ellipse, and the geodesic line becomes the focal radius vector; and, the curvature being infinite at the circumference, it passes through the other focus, and so on for ever, forming, as before, a series of angles, such that the tangents of their halves are a series of proportionals.

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* Dr. Petrie's communication will appear in full in a subsequent number of the Proceedings.